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Phase transition in Liouville theory

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We suggest that the vortices arising in a Kosterlitz-Thouless phase transition in Liouville theory correspond to transitions between different genera, producing the "plumber's nightmare" and other phases that have been predicted in fluid membrane theory from energetic considerations. This transition has previously been invoked by Cates to explain the degeneration of numerical simulations of Gaussian random surfaces into branched polymers. The difficulty in quantizing Liouville theory for $d > 1$ is conjectured to be due to our insistence on working at a fixed genus.

In a recent paper Cates[1] observed that the tendency of random surface simulations of a purely area (Gaussian) action with fixed topology to degenerate into branched polymerlike configurations[2] might be explained by a Kosterlitz-Thouless[3] (KT) phase transition in the Liouville theory associated with the surface. This phase transition, just as in the original Kosterlitz-Thouless model, is driven by vortexlike configurations called spikes. Unlike the original KT model, however, the transition does not correspond to the unbinding of charge-1 vortices, but to the appearance of charge-2 vortices. In this paper we shall examine the role of such vortex configurations when we allow the topology of our surface to vary and link the appearance of the "plumber's nightmare" and other phases of fluid membrane theory that have been predicted on energetic grounds by Huse and Leibler[4] to the vortex (high-temperature) phase of the KT model. We conjecture that the difficulty in quantizing the Liouville theory of a surface in $d>1$ is due to the unbound vortices causing transitions between different genera. We shall also examine the possible role of the plumber's nightmare in superstring theory where the extrinsic curvature[5] induced by the fermions may suppress spikes. Finally, we mention other approaches to both Liouville theory and conformal field theory on an arbitrary-genus hyperelliptic surface that suggest a KT transition may play an important role. Throughout the paper we shall appeal to random surface simulations in order to get a hint of what to expect in the continuum, whether it be in a string theory or in a fluid membrane theory.

We shall repeat Cates's arguments here for completeness. If we consider the Polyakov partition function for the bosonic string in d dimensions,[6]

$$Z=\int DX\,Dg\,\exp\left[-\frac{1}{8\pi}\int d^2x\sqrt{g}\,g^{ab}\partial_a X^\mu \partial_b X_\mu\right], \quad (1)$$

where $\mu=1,2,\ldots,d$ and $a,b=1,2$, and integrate out the X fields after choosing a conformal gauge $g_{ab}=\exp(\phi)h_{ab}$ we find

$$Z=Z'\int D_h\phi\,\exp\left[\frac{d-25}{48\pi}\int d^2x\left(\tfrac{1}{2}\sqrt{h}\,h^{ab}\partial_a\phi\partial_b\phi+\sqrt{h}\,R\,\phi\right)\right]. \quad (2)$$

In the above

$$Z'=\int\prod_i dm^i|\partial_{-1}\partial_0^{-d/2}|^2(\det \mathrm{Im}\tau)^{-d/2}, \quad (3)$$

where the m^i are the moduli of the world sheet traced out by the string, τ is the period matrix, and R is the scalar curvature for the background metric h_{ab}. The subscript on $D_h\phi$ refers to the fact that we are using a ϕ-independent measure defined by the norm

$$\|\delta\phi\|^2=\int d^2x\sqrt{h}\,(\delta\phi)^2, \quad (4)$$

and 25 rather than 26 appears in front of the ϕ, or Liouville, action because we are treating it as a quantum field.[7] The inclusion of the metric fluctuations via ϕ means that, in solid-state-physics parlance, we are discussing a fluid membrane rather than a tethered one.[8] In a random surface simulation one would represent such a fluid membrane by incorporating dynamical triangulations.[9–11] Inspired by the resemblance of the ϕ action to the KT model in the continuum, we inquire into the effects of vortexlike configurations of the form

$$\phi=-\lambda\ln(r), \quad (5)$$

where $r=\sqrt{x^2+y^2}$ and we shall call $-\lambda$ the charge of the vortex (the minus sign has been chosen to agree with Cates's notation). We have centered the vortex at the origin purely for notational convenience.

We now demand that the area around a vortex such as (5) be convergent as seen in the d-dimensional space in which the surface is embedded. We can achieve this by integrating over a small region of linear dimension a around the vortex and asking that the proper area

$$\epsilon^2\simeq\int_0^a \exp[\phi(r)]d^2x \quad (6)$$

be convergent, where we have assumed $h_{ab}\simeq\delta_{ab}$ in the region of interest. We can use the differential version of (6) to define $a(r)$ which is a local cutoff in x space. This is given in terms of the reparametrization-invariant cutoff ϵ by

$$a(r)^2\simeq\epsilon^2\exp[-\phi(r)]. \quad (7)$$

Convergence of (6) enforces $\lambda<2$. Vortices (or "spikes") with $\lambda\geq 2$ are called pathological by Cates and for these the local cutoff $a(r)$ vanishes. If we try to keep $a(r)$ finite the proper area will diverge and it is postulated that this is precisely the spiky degeneration into branched polymers which is observed in the numerical simulations of a random surface with fixed topology.[2] The partition function for these is given by the discretization of (1) at finite area,

$$Z=\sum_{\text{triang}}\int\prod_{\text{vertices}}DX_i^\mu \exp\left[-\tfrac{1}{2}\sum_{\text{edges}}(X_i^\mu-X_j^\mu)^2\right], \qquad (8)$$

where we sum over possible triangulations of the world sheet with a fixed number of vertices n in order to mimic the effects of ϕ.

We can work out the free energy of one of the spikes by following KT (Ref. 3). We keep the cutoff a corresponding to $a(r)$ at the center of the spike finite and restrict the system to be of linear size L, giving

$$U=\frac{25-d}{48}\lambda^2\ln\left[\frac{L}{a}\right], \qquad (9)$$

for the energy. The positional entropy, on the other hand, is given by

$$S=2\ln\left[\frac{L}{a}\right], \qquad (10)$$

so the free energy is

$$F=U-S=\left[\frac{25-d}{48}\lambda^2-2\right]\ln\left[\frac{L}{a}\right]. \qquad (11)$$

Pathological spikes will thus dominate the partition function when $d>1$. This is the region where the current quantizations of Liouville theory break down[12,13] so Cates suggests that the strong-coupling regime of Liouville theory, $1<d<25$, corresponds to the vortex phase of the KT model.

The effect of the vortices on the critical theory in 26 dimensions is not clear. The infinite-temperature limit of the KT model corresponds to $d\to25$ from below, but Das, Naik, and Wadia[14] suggest that it is natural to define the critical theory by taking $d\to25$ from *above* and making the analytically continued Liouville mode $\phi\to i\phi$ take the part of X^0. The theory in this regime would lie outside the scope of the preceding discussion.

We now observe that if we consider (11) at finite a, vortices with $\lambda=-2$, i.e.,

$$\phi=2\ln(r), \qquad (12)$$

also dominate the partition function when $d>1$. There is, however, no divergence of proper area associated with such a positively charged vortex, as consideration of (6) reveals. They do not, therefore, correspond to a spiky degeneration. Cates envisions his spikes being stabilized against dissociation into infinitesimal charges by the singularity at their core, calling them a strictly nonperturbative phenomenon. We do not have this luxury for our charge $+2$ vortices, so if they are to play a role in the continuum theory we must suggest some way of stabilizing them.

We can obtain a hint as to what might do this by remembering that we are working in a conformal gauge, so

$$\sqrt{g}R_{\text{total}}=\Delta\phi+\sqrt{h}R, \qquad (13)$$

where R_{total} is the curvature scalar of the full metric g_{ab} and Δ is the scalar Laplacian. The Riemann-Roch theorem tells us that

$$\frac{1}{4\pi}\int d^2x\sqrt{g}R_{\text{total}}=2-2n, \qquad (14)$$

where n is the number of handles, or genus, of the surface. The quantity on the right-hand side of (14) is just the Euler characteristic χ. The appearance of a charge-2 vortex such as (12) will decrease χ by 2, which would also be the result of adding a handle if we allowed changes in topology. In Cates's paper the charge -2 vortices cannot be thought of as corresponding to the removal of handles because there the topology was fixed by fiat, as it is in many of the numerical simulations. For a fixed topology one could maintain the same χ upon the addition of a vortex if one also added a smeared-out countercharge. In view of the preceding discussion we would claim, with the same amount of rigor as Cates, that our vortices were stabilized by being associated with a change in topology

When we draw the parallel between adding a handle and the appearance of a charge $+2$ vortex we have in mind the following picture. We adopt a Mandelstam, or light-cone,[15] parametrization for the metric g_{ab} because this is ideally suited to accommodating the inclusion of configurations that have singular curvatures, such as vortices. The Mandelstam parametrization makes use of the unique meromorphic one-form ω_z that has poles at any punctures P_r in the surface, with residues α_r satisfying

$$\sum_r\alpha_r=0, \qquad (15)$$

and which has purely imaginary periods around any handles. (15) restricts us to surfaces with two or more punctures. This is not necessarily unphysical when we are thinking of real membranes rather than string world sheets. For example, the n-tachyon amplitude for a bosonic string corresponds to evaluating the partition function for a surface that has been pinned at n points. The position space representation of the tachyon vertex operator makes this most transparent, because we can see it corresponds to pinning the surface at an arbitrary point X_i (Ref. 16):

$$O_i(X)=\int d^2x\sqrt{g}\,\delta^d(X^\mu(x)-X_i^\mu). \qquad (16)$$

The insertion points of the vertex operators thus provide us with the punctures required in order to define ω_z. More recently Mandelstam has suggested that one can choose a *holomorphic* one-form rather than ω, which would allow the consideration of unpunctured surfaces.[17] In any case, given ω or its holomorphic counterpart we can set

$$g_{z\bar z}=\omega_z\omega_{\bar z}, \qquad (17)$$

where we have lapsed into complex notation. The curvature in such a parametrization is concentrated at the poles P_r and zeros R_a of ω:

$$\sqrt{g}R_{\rm total}=-2\pi\left[\sum_{a=0}^{2n-3+p}\delta^2(z-R_a)-\sum_{r=0}^{p}\delta^2(z-P_r)\right], \tag{18}$$

where we have written the sums for a genus n surface with p punctures. The zeros of ω are at the interaction points of the Mandelstam diagram and the poles at the punctures associated with the vertex operator insertions. The concentration of curvature in (18) at delta-function singularities is precisely why the Mandelstam parametrization is suited to the inclusion of vortices.

We now consider what the Liouville theory will look like in the presence of a vortex. In order to accommodate the vortex we shall expand ϕ about it to get

$$\phi=-\lambda\ln(r)+\phi' , \tag{19}$$

where ϕ' is our new Liouville field. The vortex will contribute to $R_{\rm total}$,

$$\sqrt{g}R_{\rm vortex}=-2\pi\left[\sum_{a=0}^{2n-3+p}\delta^2(z-R_a)-\sum_{r=0}^{p}\delta^2(z-P_r)-\lambda\delta^2(z)\right], \tag{20}$$

acting like the addition of an infinitesimal handle at $z=0$ if it has charge $+2$. The charge -2 vortex could be interpreted as the nucleation of a droplet from the surface or the destruction of an already existing infinitesimal handle. We might inquire as to why charge-1 vortices, which by (11) would disorder the system for $d>-71$, do not appear. In the Mandelstam parametrization a handle has a positive charge at either end so the total charge associated with it will always be $+2$. A puncture has charges of ±1 associated with it, but the appearance of such a vortex pair requires the insertion of a vertex operator to create the puncture, in other words, the presence of an external excitation. In the absence of such an excitation charge ±1 vortices will be suppressed.

The above description of the Liouville action is that of a Coulomb gas of curvature singularities. We can see this more explicitly by making use of the results of D'Hoker and Phong[18] which give the Liouville action for a metric of the form (17). They consider a holomorphic one-form, but their results also apply to the meromorphic case. The Liouville action may be written in terms of the Arakelov Green's function $G^A(z,w)$ for $h_{z\bar z}$, which satisfies

$$-\partial_z\partial_{\bar z}G^A(z,w)=2\pi\delta(z,w)+\frac{h_{z\bar z}R}{2(n-1)} \tag{21}$$

and

$$\int d^2z\, g_{z\bar z}RG^A(z,w)=0 . \tag{22}$$

They find that

$$S_{\rm Liouville}=\frac{25-d}{24}\sum_{a,b=1}^{2n-2}q_aq_bG^A(R_a,R_b)+\frac{25-d}{6}(n-1)c , \tag{23}$$

where c is a constant measuring the relative normalizations of $\omega_z\omega_{\bar z}$ and $h_{z\bar z}$. If we have a holomorphic ω, as above, all the q_a are -1. The asymptotic behavior of $G^A(z,R_a)$ is just

$$G^A(z,R_a)\simeq-\ln\|z-R_a\|^2 , \tag{24}$$

so we have arrived at our promised Coulomb-gas representation. The q_a for a vortex such as (5) is given by λ. If the theory undergoes a phase transition at $d=1$ we can see that charges of ±2 will be responsible, as indicated by our earlier more heuristic arguments.

To proceed further we shall consider the effect of our vortices in a numerical simulation. In a typical random surface simulation we have a partition function of the form

$$Z=\sum_{c\in C}\exp[-A(c)+\mu\chi(c)] , \tag{25}$$

where $A(c)$ is the area of the surface which is just given by the usual bosonic string action. The surface is allowed to vary over some set of allowed configurations C. We could decompose (25) into a sum over surfaces of different genus to get

$$Z=\sum_{n=-N}^{N}e^{2\mu}e^{-2\mu n}Z_n , \tag{26}$$

where Z_n is given by (1) evaluated on the appropriate surface of genus n. We have allowed disconnected components which gives the lower bound on n as $-N$ rather than zero. The magnitude of N would be determined by the size of our lattice (the finer the lattice, the more handles or droplets we could fit onto it). The parameter μ acts as a chemical potential for the Euler characteristic of the surface and $e^{-2\mu}$ is effectively the string coupling. For large positive μ, χ will be driven large and negative, which would correspond to a preponderance of positively charged vortices in our Mandelstam picture, producing a surface with a large number of handles. In a theory in which a two-dimensional fluid membrane was embedded in three-dimensional space this could give rise to a spongelike phase, which is the plumber's nightmare (see Fig. 1). There are many possible plumber's nightmares depending on the symmetry of the unit cell and its stacking. The details of the interactions in the fluid membrane theory will determine which one appears. Large negative μ, on the other hand, will favor the formation of a phase in which the ground state is composed of a large number of disconnected droplets. At intermediate μ the system may be in one of the other phases discussed by Huse and Leibler, such as the random isotropic phase.

If our surface is embedded in $d>1$ dimensions we have argued that the Liouville theory is in a vortex phase so it will be free to explore the configurations described above. The free energy will favor the formation of vortices, in other words, handles or droplets, so it would seem most

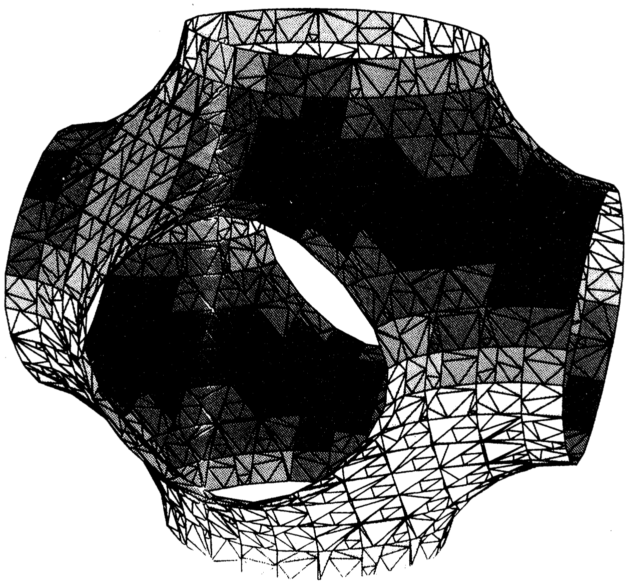

FIG. 1. The unit cell of a simple cubic plumber's nightmare. These stack together horizontally and vertically to give the plumber's nightmare.

natural to consider an ensemble in which the topology of the surface is allowed to change for the quantization. The above observations do not apply exclusively to random surfaces or strings. We would expect the Liouville theory coupled to any $c > 1$ conformal field theory to be in the vortex phase, so we would see transitions between genera in the dynamically triangulated lattice one would use in simulating such a theory. The mean value of χ will be determined by μ, but the surface will be free to fluctuate about it. A value close to zero would not necessarily mean that the system was fluctuating around something with the topology of, say, a sphere or torus, it could be composed of one component with many handles and a large number of droplets.

Going back to the continuum, the preceding discussion suggests strongly that the failure of the quantized Liouville theory for $d > 1$ is due to our insistence on working at a fixed genus. The standard approach to a conformal field theory starts with a sphere and works up from there, but the picture of unbound vortices means that transitions between different topologies should be taken into account. It is perhaps worth remarking at this point that the volume of moduli space for a genus-n surface goes as $n!$ (Ref. 19) so, in a string model at any rate, the tendency would be to run off to infinite genus because of the greater phase-space volume with increasing genus. For a lattice simulation the size of the lattice would provide a cutoff, as would the size of the constituents for a real fluid membrane. Huse and Leibler consider the free energy per unit volume for a plumber's nightmare and find that it can be made arbitrarily negative by taking the size of the unit cell to zero, providing evidence for the above behavior.

It seems that we have here a case where a quantum-gravity theory (in this case the Liouville theory) demands the inclusion of topology changing transitions, which should be music to the ears of the practitioners of wormhole physics. Indeed, Polchinski[20] has suggested that Liouville theory coupled to matter with c *greater* than 25 may be a good toy model for understanding wormhole effects in quantum gravity. (The constraint on c was chosen so that the Liouville theory with a positive two-dimensional cosmological constant had de Sitter-type solutions.)

Let us consider the effects of introducing such a two-dimensional cosmological constant Λ in our discussion:

$$S = S_{\text{Liouville}} + \int d^2x \sqrt{g}\, \Lambda \ . \tag{27}$$

Cates proposes two possible forms of behavior upon the addition of Λ at a fixed topology. Either the density of spikes for $d > 1$ could be controlled continuously by Λ, or there could be a first-order transition as a function of Λ to a spiky phase. We now consider what could happen if we relax the constraint of fixed topology. The equation of motion for the Liouville theory with a cosmological constant is

$$\frac{d-25}{48\pi} R_{\text{total}} = \Lambda \ , \tag{28}$$

and in $d=3$ we have

$$R_{\text{total}}(x) = \frac{1}{r_1(x) r_2(x)} \ , \tag{29}$$

where r_1 and r_2 are the radii of curvature of the surface embedded in three-space. On a plumber's nightmare where the handles have a characteristic size r, $r_1(x) = -r_2(x) = r$ everywhere (we are neglecting the boundaries, if any, of the surface), so it could be a solution of the equation of motion for positive Λ. r would presumably be of the order of the proper time cutoff ϵ if we imposed a cutoff on the theory. Given the tendency to run off to the maximum possible genus the ground state of the theory might look like a plumber's nightmare at every scale, provided of course that the form of the effective action from scale to scale was the same.

Configurations such as the plumber's nightmare may also have a role to play in superstring theory. Wiegmann has recently shown that integrating out the fermions in either a Neveu-Schwarz-Ramond (NSR) or Green-Schwarz superstring[21] gives

$$S_{\text{eff}} = W_1(A^n) + \frac{\tau_\lambda}{8} \int d^2x [(\mathbf{e}_a \partial_b \mathbf{e}_c)^2 + (\nabla_a \mathbf{n}_i)^2] \ , \tag{30}$$

where the $\mathbf{n}_i$ are the normals to the surface, the $\mathbf{e}_a$ its tangents, and we have dropped the bosonic and gauge fermion terms. The Wess-Zumino action is given by

$$W_k(A) = \frac{ik}{8} \text{tr} \left[\tfrac{1}{2} \int d^2x \ A \wedge A \right.$$
$$\left. + \frac{1}{3\pi} \int_D d^3x \ A \wedge A \wedge A \right] \ , \tag{31}$$

where D is a three-dimensional disk bounded by the world sheet. τ_λ is a Dynkin factor that depends on the representation for the fermions and

$$A^n = A^n_{ij} M_{ij}, \quad A^n_{ij} = \mathbf{n}_i d\mathbf{n}_j , \tag{32}$$

where M_{ij} is a generator of $SO(d-2)$. The heterotic differs only in the Wess-Zumino term. We see that the fermions have generated a Wess-Zumino action, a term in the tangent vectors, and the integrated extrinsic curvature squared:

$$\int d^2x\, K^2 = \int d^2x (\nabla_a \mathbf{n}_i)^2 . \tag{33}$$

It is known from numerical simulations that the extrinsic curvature is an effective spike killer for fixed topologies, so the addition of fermions may suppress the spikes by inducing just such a term. Thus, even though the super-Liouville theory is in a strong-coupling, or vortex, regime for $1<d<9$ one might think that smooth configurations of the world sheet could still predominate. However, a plumber's nightmare would not be suppressed by (33) as it does not have any extrinsic curvature, for the three-dimensional case at any rate (nobody has considered whether the equivalent of the plumber's nightmare exists in more than three dimensions). We can see this by noting

$$K(x) = \frac{1}{r_1(x)} + \frac{1}{r_2(x)} , \tag{34}$$

where $r_1(x)$ and $r_2(x)$ are the two radii of curvature we defined earlier. If we allowed topological transitions as we have argued one should, such a configuration could arise, causing the breakdown of the super-Liouville theory for an NSR string. The ordinary Liouville theory associated with a subcritical Green-Schwarz string, which could be defined in three, four, or six dimensions,[22] could similarly accommodate a plumber's nightmare in the three-dimensional case.

Once again, the role of vortex configurations in the critical theory is not clear. For the NSR string one might be tempted to define the critical theory by letting $d \rightarrow 9$ from above and using $\phi \rightarrow i\phi$ and $\chi \rightarrow i\chi$, where we have gauge fixed the gravitino by $\chi_a = \sigma_a \chi$, as X^0 and ψ^0, respectively. This would take the theory outside the scope of our discussion, just as in the bosonic case. This option is not open to us for the Green-Schwarz string, because there the spacetime supersymmetry would be lost outside three, four, or six dimensions. Perhaps demanding spacetime supersymmetry (in the NSR case by summing over spin structures) could protect the world sheet from topological transitions.

We might also observe that although the addition of an extrinsic curvature will prevent the collapse of a random surface simulation to a branched polymer at fixed topology it will not prevent the occurrence of a plumber's nightmare for positive μ or a lamellar phase for negative μ if the topology is allowed to change. Regarded purely as a random surface simulation there is no reason to object to this, but if we are thinking of defining a string theory as the continuum limit of such a theory this might be more worrisome. It would certainly be interesting to carry out a simulation incorporating area, extrinsic curvature, and Euler characteristic terms and allowing topology changes to explore the rich phase structure that is predicted by the work of Huse and Leibler.

It would be worthwhile reconciling the picture here with that of Foerster,[23] who discretized Liouville theory and found that its failure at $d>1$ was due to the impossibility of maintaining conformal invariance. He used infinitesimal curvature charges rather than the singular finite charges appearing in the light-core parametrization. Our discussion also has some similarities to that of La,[24] who considered free fermions on a hyperelliptic surface and discovered a KT-type transition. In his approach the vortices corresponded to the branch-point operators and their dissociation to the loss of the notion of genus. In our case, rather than having the bound states corresponding to handles dissociating, we have handles appearing and disappearing, so the notion of genus is not lost entirely. The light-cone parametrization we use has the advantage of providing a cover of all of moduli space, which the hyperelliptic approach fails to do for genus >2. Whichever of these descriptions finally proves the most useful it is clear that the investigation of topological transitions in Liouville theory is worthy of further attention.

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